Key 2019

Math 10C

Final Review

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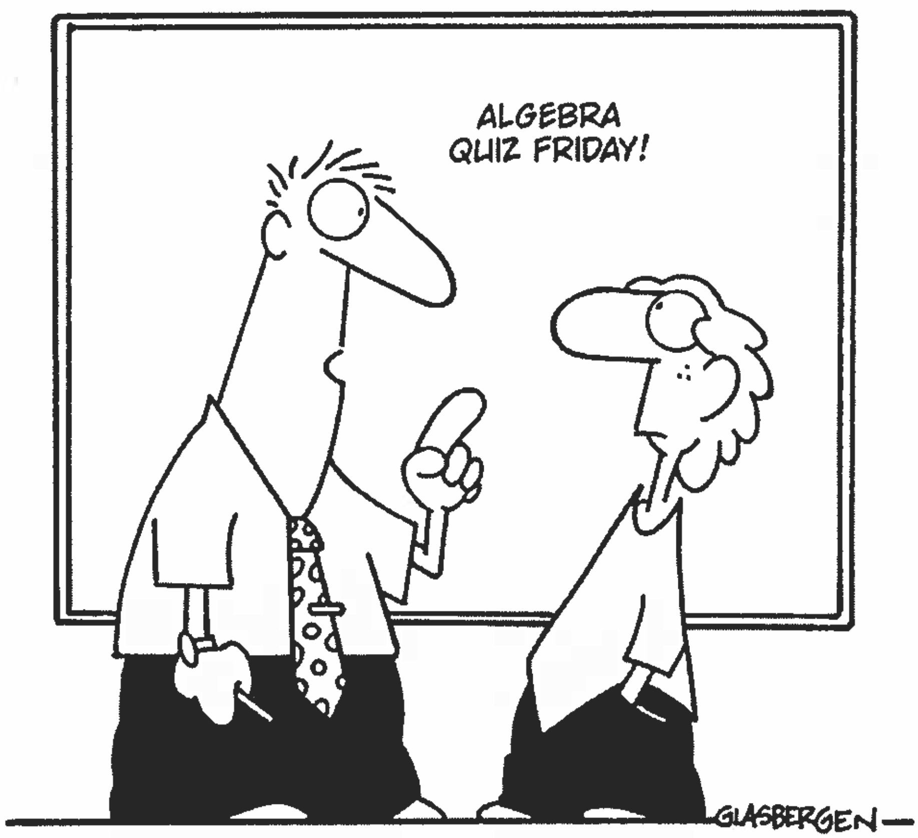


"It's important to learn math because someday you might accidentally buy a phone without a calculator."

Unit 1: Number

Prime Number: a whole number with exactly two factors (ex: 1, 3, 7, 19)
Composite Number: a whole number with more than two factors (ex: 25, 78, 100)
Prime Factors: factors of a number which are prime
Prime Factorization: expressing a number as the product of prime factors

Tree Diagram

$100 = 2^2 \cdot 5^2$

100
2 50
5 10
5 2

• answers will vary.

Greatest Common Factor (GCF): largest whole number which divides exactly into each of the members of the set
Lowest Common Multiple (LCM): lowest multiple common between the members of the set

Ex: Determine the GCF and LCM of 35, 231, and 275

35: 7 5

231: 3 77; 77: 7 11

275: 5 55; 55: 5 11

nothing in common.

$GCF = 1$

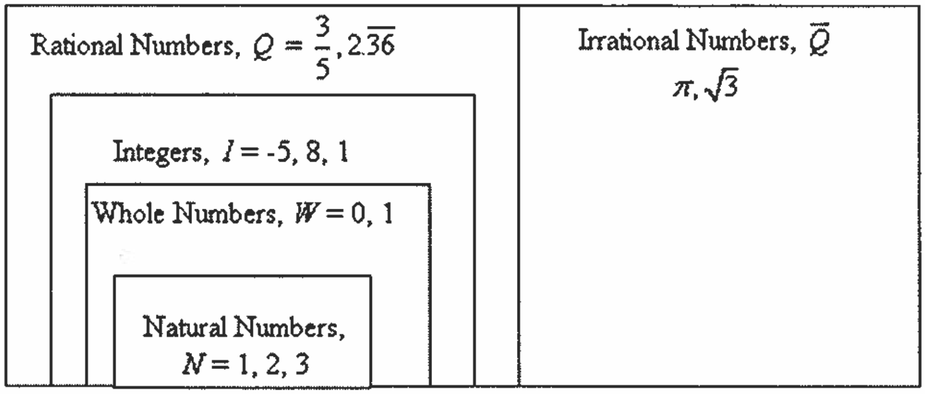
$LCM = 3 \cdot 7 \cdot 5^2 \cdot 11$
$= 5775$

Rational Numbers: can be written as a ratio of two integers → repeat or terminate → can be converted into fractions
Irrational Numbers: both non-repeating and non-terminating → cannot be converted into fractions

Ex: algebraically and graphically convert $0.\overline{36}$ to a fraction in lowest terms

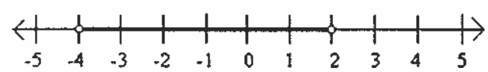
$$\frac{36}{99} = \frac{4}{11}$$

Real Number System: Fractions No Fractions

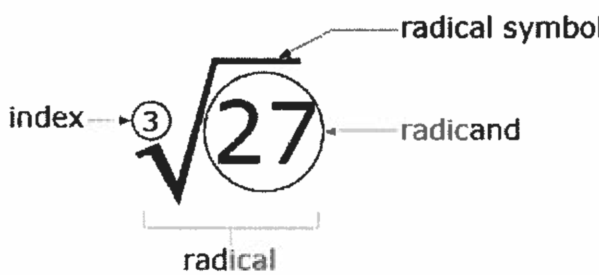


Absolute Value Inequalities:

$x \leq -2$ or $x \geq 3$ $-4 < x < 2$



Parts of a Radical



The product/quotient of the roots of two numbers is equal to the root of the product/quotient of the two numbers.
The sum/difference of the roots of two numbers is **NOT** equal to the root of the sum/difference of the two numbers.

Ex.

$$\sqrt{9} \times \sqrt{4} = \sqrt{9 \times 4} = \sqrt{36} = 6$$
$$\sqrt{9} + \sqrt{4} \neq \sqrt{9+4}$$

Ex: Convert mixed radicals to entire radicals.

$2\sqrt{192}$ $\quad$ $3\sqrt[3]{2}$ $\quad$ $\frac{3}{4}\sqrt{160}$ $\quad$ $\sqrt{\frac{7}{9}}$

$\sqrt{2^2 \cdot 192}$ $\quad$ $\sqrt[3]{3^3 \cdot 2}$ $\quad$ $\sqrt{160\left(\frac{3}{4}\right)^2}$ $\quad$ $\sqrt{\frac{7}{9}}$

$\sqrt{768}$ $\quad$ $\sqrt[3]{54}$ $\quad$ $\sqrt{90}$

Ex: Convert entire radicals to mixed radicals in simplest form

$\sqrt[3]{48}$

$48 \to 2,\ 24 \to 2,\ 12 \to 2,\ 6 \to 2,\ 3$

$\sqrt[3]{8\cdot 6}$

$= 2\sqrt[3]{6}$

$\sqrt[3]{-16}$

$-16 \to -2,\ 8 \to 2,\ 4 \to 2,\ 2$

$= \sqrt[3]{-8\cdot 2}$

$= -2\sqrt[3]{2}$

$\sqrt{75}$

$75 \to 3,\ 25 \to 5\ 5$

$= \sqrt{25\cdot 3}$

$= 5\sqrt{3}$

$11\sqrt{242}$

$242 \to 2,\ 121 \to 11\ 11$

$= 11\sqrt{121\cdot 2}$

$= 11\cdot 11\sqrt{2}$

$= 121\sqrt{2}$

Unit 2: Exponents

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad (a \neq 0)$$

$$(a^m)^n = a^{mn}$$

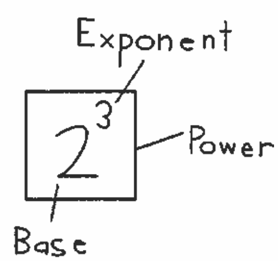
$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^0 = 1 \qquad (a \neq 0)$$

$$a^{-n} = \frac{1}{a^n} \qquad (a \neq 0)$$

Really understand these rules



Examples:

$$\frac{10e^8 f^{12}}{4e^4 f^7} = \frac{5e^4 f^5}{2}$$

$$\frac{x^{5a+7b}\cdot x^{3a+b}}{x^a \cdot x^{2a-7b}} = \frac{X^{8a+8b}}{X^{3a-7b}} = X^{5a+15b}$$

$$5x^3y^{-8}z^{-2} \div \frac{15x^8y^3z^{-1}}{x^5y^{-3}z^2}$$

$$\frac{5x^3}{y^8z^2} \cdot \frac{x^5y^{-3}z^2}{15x^8y^3z^{-1}}$$

$$\frac{5\cancel{x^8}y^{-3}z^2}{15\cancel{x^8}y^{11}z^1}$$

$$\frac{z}{3y^{14}}$$

Ex: Write in radical form then evaluate.

$$16^{\frac{-3}{4}} = \left(\frac{1}{16}\right)^{3/4} = \frac{\left(\sqrt[4]{1}\right)^3}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\left(\frac{9}{4}\right)^{\frac{3}{2}} = \frac{\left(\sqrt{9}\right)^3}{\left(\sqrt{4}\right)^3} = \frac{3^3}{2^3} = \frac{27}{8}$$

Ex: Write an equivalent expression using exponents.

$\sqrt{a^3} = a^{3/2}$

$\sqrt[3]{64v^6}$

$(64v^6)^{1/3}$

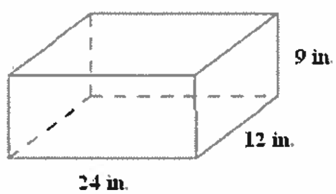
$\left(\sqrt[4]{x^5y^3}\right)^{\frac{3}{2}}$

$\left(\left(x^5y^3\right)^{\frac{1}{4}}\right)^{\frac{3}{2}}$ OR

$\left(x^5y^3\right)^{\frac{3}{8}}$

Unit 3: Measurement

Determine the surface area and volume measurements.



$SA = 2(9\times12) + 2(24\times12) + 2(9\times24)$

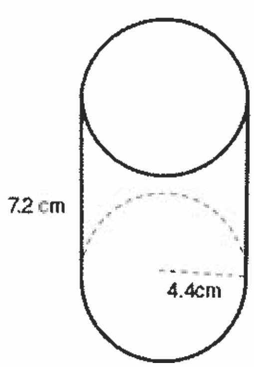
$= 216 + 576 + 432$

$= 1224 \text{ in}^2$

$V = 24\times12\times9$

$= 2592 \text{ in}^3$

Calculate the surface areas and volumes.



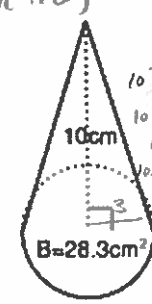
$SA = 2\pi(4.4)^2 + 2\pi(4.4)(7.2)$

$= 121.6 + 199.05$

$= 320.7 \text{cm}^2$

$V = \pi(4.4)^2(7.2)$

$= 437.9 \text{cm}^3$



$10^2 + 3^2 = s^2$

$100 + 9 = s^2$

$109 = s^2$

$10.4 = s$

$A = \pi r^2$

$\frac{28.3}{\pi} = \frac{\pi r^2}{}$

$9.008 = r^2$

$3 = r$

$SA = \pi(3)^2 + \pi(3)10$

$= 28.27 + 98.39$

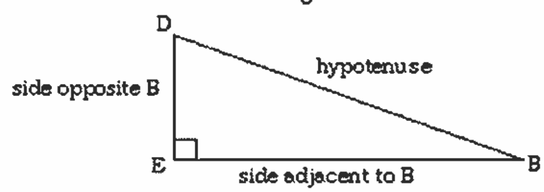
$= 126.66 \text{ cm}^2$

$V = \frac{1}{3}\pi(4.4)^2(10)$

$= 202.7 \text{ cm}^3$

Unit 4: Trigonometry

The Basic Trigonometric Ratios



$\sin B = \frac{opp}{hyp}$

$\cos B = \frac{adj}{hyp}$

$\tan B = \frac{opp}{adj} = \frac{\sin B}{\cos B}$

Abbreviations for side lengths:

opp : opposite
adj : adjacent
hyp : hypotenuse

Figure 6.9

Calculate x:

$\sin x = \frac{4}{5}$ $x = 53°$

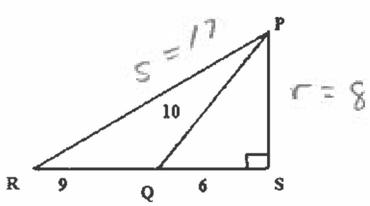
$\cos x = \frac{2.3}{4.5}$ $x = 59°$

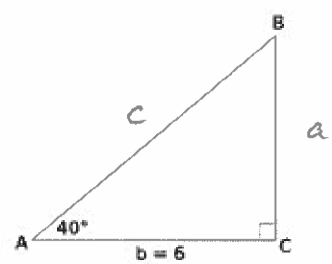
* inverse when finding an angle

$\tan 36° = x$ $x = 0.7265$

$\sin 78° = x$ $x = 0.9781$

Solve:





$10^2 - 6^2 = r^2$

$15^2 + 8^2 = s^2$

$\angle R \Rightarrow \tan^{-1}(8/15)$
$= 28°$

$\angle Q \Rightarrow \tan^{-1}(8/6)$
$= 53°$

$\angle P \Rightarrow \tan^{-1}(15/8)$
$= 62°$

$\angle B = 50°$

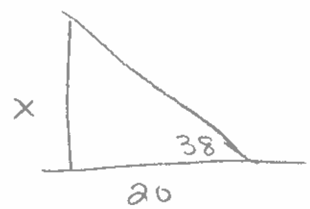
$\tan 40 = \frac{a}{6}$

$a = 5.0$

$\cos 40 = \frac{6}{c}$

$c = 7.8$

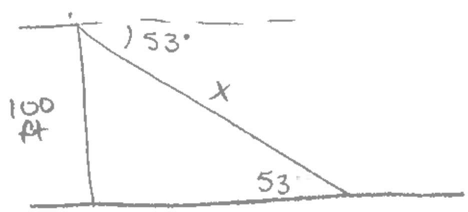
You are standing 20 feet away from a tree, and you measure the angle of elevation to be 38°. How tall is the tree?



$\tan 38 = \frac{x}{20}$

$x = 15.6 \text{ ft}$

You are standing on top of a building, looking at park in the distance. The angle of depression is 53°. If the building you are standing on is 100 feet tall, how far away is the park? Does your height matter?



$\sin 53 = \frac{100}{x}$

$x = 125.2 \text{ ft}$

yes, it matters
you need at
least one
length to find
another length.

Unit 5: Polynomial Operations

Monomial: number, variable, or the product of a number and a variable → only one term
Binomial: two terms
Trinomial: three terms
Polynomial: monomial or sum or difference of monomials → exponents on the variables must be positive integers
Degree of a Monomial: sum of the exponents of its variable(s)
Degree of a Polynomial: degree of the term with the highest degree
Constant Term: term in a polynomial that has no variable
Leading Coefficient: the coefficient of the term with the highest power of the variable
Like Terms: terms with same variable raised to the same exponent
Unlike Terms: terms with different variables or the same variable raised to different exponents

Example: Expand and simplify.

a) $3(x + 5) - 7x(2x^2 - x)$

$3x + 15 - 14x^3 + 7x^2$

$-14x^3 + 7x^2 + 3x + 15$

b) $20x^3y^3 - 4x^3y^2(3x + 5y - xy)$

$\cancel{20x^3y^3} - 12x^4y^2 \cancel{- 20x^3y^3} + 4x^4y^3$

$-12x^4y^2 + 4x^4y^3$

c) $(x + 4)(2x - 1)$

$2x^2 - x + 8x - 4$

$= 2x^2 + 7x - 4$

d) $(4a - 3b)^2$ $(4a - 3b)$

$16a^2 - 12ab - 12ab + 9b^2$

$= 16a^2 - 24ab + 9b^2$

e) $(3x - 1)(2x + 5) - 5(8x + 3)(2x - 7)$

$6x^2 + 15x - 2x - 5 - 5[16x^2 - 56x + 6x - 21]$

$6x^2 + 13x - 5 - 5(16x^2 - 50x - 21)$

$- 80x^2 + 250x + 105$

$= -74x^2 + 263x + 100$

f) $(x^2 - 7)(4x^2 - 3x - 1)$

$4x^4 - 3x^3 - x^2 - 28x^2 + 21x + 7$

$= 4x^4 - 3x^3 - 29x^2 + 21x + 7$

Unit 6: Factoring Polynomial Expressions

Factoring: write the sum or difference of monomials as a product of polynomials

Rules: **1.** Factor out the Greatest Common Factor from the polynomial
2. Difference of Squares?
3. Determine the sum and product integers. (a, b)
4. No leading coefficient: $(x + a)(x + b)$
Leading coefficient: method of decomposition.
5. Check by expanding.

Note: to find the sum and product: if the product is positive, integers must either both be positive or both be negative, depending on the sum. If the product is negative, there must be one positive and one negative integer.

Examples: Factor completely.

a) $x^2 + 8x + 12$

$= (x + 6)(x + 2)$

b) $3x^3 + 21x^2 + 30x$

$3x(x^2 + 7x + 10)$

$= 3x(x + 5)(x + 2)$

c) $x^2 - x - 12$

$= (x - 4)(x + 3)$

d) $3a^2 - 15ab - 252b^2$

$3(a^2 - 5ab - 84b^2)$

$= 3(a - 12b)(a + 7b)$

e) $x^2 - 49$

$= (x-7)(x+7)$

f) $25x^2 - 64y^2$

$= (5x-8y)(5x+8y)$

g) $24x^2 - 90x + 54$

$6(4x^2-15x+9)$

$6(\underbrace{4x^2-12x}_{4x}\,|\,\underbrace{-3x+9}_{-3})$

$4x(x-3)-3(x-3)$

$= 6(4x-3)(x-3)$

h) $6 - 7x - 20x^2$

$-20x^2-7x+6$

$-1(20x^2+7x-6)$

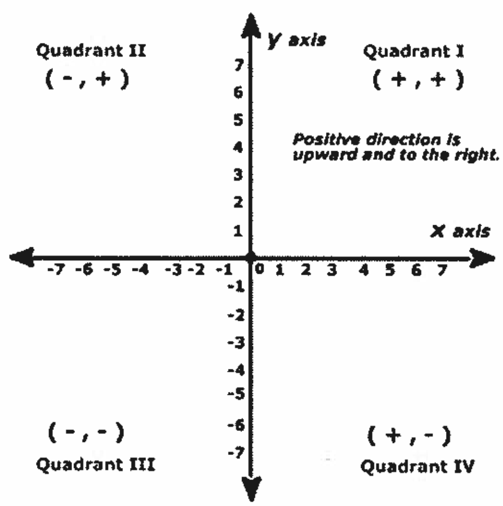
$-1(\underbrace{20x^2+15x}_{5x}\,|\,\underbrace{-8x-6}_{-2})$

$5x(4x+3)-2(4x+3)$

$= -1(5x-2)(4x+3)$

Unit 7: Relations and Functions

Origin: usually labeled O, points (0, 0)
Ordered Pair: a specific point on a Cartesian plane. The numbers in the ordered plane are called coordinates.
Coordinates: x-coordinate and y-coordinate make up an ordered pair → can be plotted on a Cartesian plane



Discrete Variable: can only take on limited values
Continuous Variable: can take on every value within a particular interval
Relation: a comparison between two sets of elements
Dependent Variable/Output/Range: y → vertical axis → second coordinate
Independent Variable/Input/Domain: x → horizontal axis → first coordinate
Y-Intercept: y-coordinate of the ordered pair where the graph intersects the y-axis → where x=0
X-Intercept: x-coordinate of the ordered pair where the graph intersects the x-axis → where y=0

Example: Determine the x and y intercepts of the equation $3y = 5x + 15$

xint $y=0$ $(-3,0)$ yint $x=0$ $(0,5)$

$\cancel{3}(0) = 5x + 15$

$-15 = 5x$

$-3 = x$

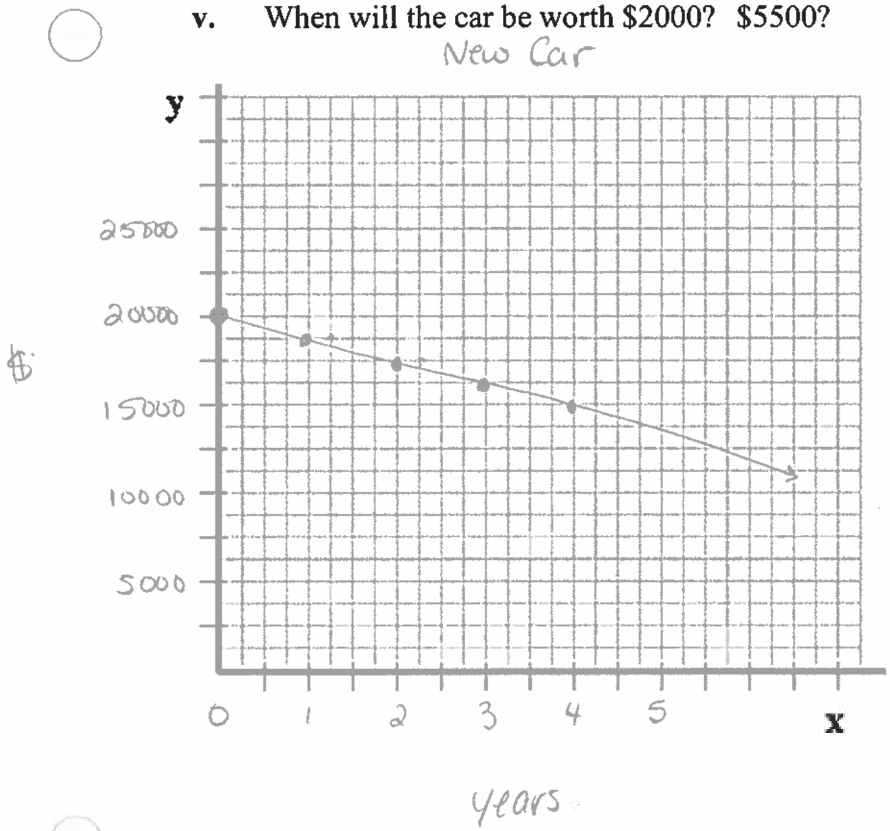
$\frac{3y}{3} = \frac{5(0) + 15}{3}$

$y = 5$

Interpolation: using the graph to find values lying between given points
Extrapolation: extending the graph to predict values outside the plotted points

Example: Johnny purchases a new car for \$20 000. The value of the car can be represented by the formula V = 20 000 – 1250t, where V is the value of the car in dollars, and t is the age of the car in years.

- **i.** Complete a table of values up to 4 years and plot them on the grid.
- **ii.** What does the ordered pair (0, 20 000) represent?
- **iii.** Calculate the t-intercept and determine what this number represents.
- **iv.** Calculate the value of the car after 3 years and 12 years.
- **v.** When will the car be worth \$2000? \$5500?



i)

x	y
0	20000
1	18750
2	17500
3	16250
4	15000

ii) yintercept, the value of the car was initial \$20000

iii) $0 = 20000 - 1250t$

$1250t = 20000$

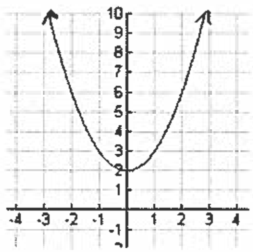
$t = 16$yrs

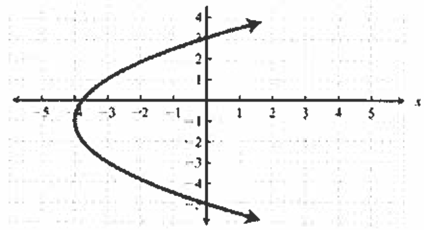
take 16 yrs till the car is worth nothing

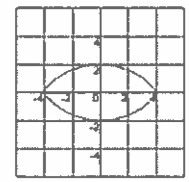
iv) \$16250 & \$5000

v) 14.4yrs + 11.6yrs.

Examples: Calculate the domain and range.







D: $\{x \in R\}$

R: $\{y \geq 2, y \in R\}$

D: $\{x \geq -4, x \in R\}$

R: $\{y \in R\}$

D: $\{-4 \leq x \leq 4, x \in R\}$

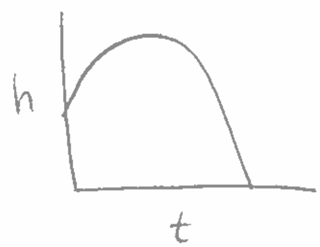
R: $\{-2 \leq y \leq 2, y \in R\}$

Example: The height of a human cannon ball, "Cano", can be described by the formula $h(t) = 12 + 6t - t^2$, where $h(t)$ is the height in metres above ground level, and t is the time in seconds. Cano is projected out of a cannon from the top of a building and lands on a soft mat. The mat is placed in a hole in the ground so that the top of the mat is level with the ground.

bonus question

a) Graph the function and sketch

only quadrant I
b/c no negative
time & height



b) Write down appropriate window you used x[0, 12, 1] y[0, 25, 1]

c) What is the height of the cannon above the ground?

t=0 means y int

12m

d) How high is cano one second after he is launched?

$h(1) = 12 + 6(1) - (1)^2$

$= 17m$

e) Write an appropriate domain and range for this relation.

D: $0 \leq x \leq$ 2nd xint (7.6)

R: $0 \leq y \leq$ max yvalue (21)

Function: a special type of relation in which each element of the domain is related to exactly one element of the range. Remember: Vertical Line Test → only one x for every y

Unit 8: Characteristics of Linear Relations

$$Slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Determine if ΔABC is a right triangle if A (0, 1), B (-3, -3), and C (-7, 0).

$m_{AB} = \frac{-3-1}{-3-0} = \frac{-4}{-3} = \frac{4}{3}$ $\quad m_{BC} = \frac{0-^{-}3}{-7-^{-}3} = \frac{3}{-4}$

$m_{AC} = \frac{0-1}{-7-0} = \frac{-1}{-7} = \frac{1}{7}$ $\quad AB \perp BC \therefore$ yes its a right triangle.

Slope: of a line segment is the measure of the steepness
Rise: change in vertical height between endpoints
Run: change in horizontal length between endpoints

Note: Horizontal line segments have a slope of 0 and vertical line segments have a slope that is undefined.

A line that rises from left to right has a **positive slope.** A line that falls from left to right has a **negative slope.**

Example: Determine the slope of PQ when P(4, 7) and Q(12, 3).

$$m = \frac{3-7}{12-4} = \frac{-4}{8} = -\frac{1}{2}$$

Example: A line segment has a slope of $-\frac{5}{7}$ and a rise of 12. Calculate the run.

$-\frac{5}{7} = \frac{12}{x}$ $\quad \frac{-5x = 84}{-5}$ $\quad x = -16.8$

Collinear: points that lie on the same line → have same slopes
Parallel: line segments that have the same slope
Perpendicular: line segments are negative reciprocals of one another → product of both slopes is -1
Example: Determine the parallel and perpendicular slopes of a line segment with points A(3, 7) and B(9, 2).

$m = \frac{2-7}{9-3} = \frac{-5}{6}$ → $\parallel \frac{-5}{6}$

↘ $\perp \frac{6}{5}$

Unit 9: Equations of Linear Relations

Linear Equation: an equation of the form $y = mx + b$ where m is the slope and b is the y-intercept. The graph of a linear equation is a straight line

Slope Y-Intercept Form: $y = mx + b$

Example: Write an equation of a line with point (0, 2) and slope $\frac{5}{2}$.

$y = \frac{5}{2}x + 2$

Example: Write and equation of a line passing through the points (0, 9) and (11, 14)

$m = \frac{14-9}{11-0} = \frac{5}{11}$ $y = \frac{5}{11}x + 9$

Standard/General Form: Ax + By + C = 0 → positive A,B, and C values → no fractions

Example: Determine the slope of the line 2x – 5y + 3 = 0

$+5y$ $+5y$

$\frac{5y = 2x+3}{5}$

$y = \frac{2}{5}x + \frac{3}{5}$

$m = \frac{2}{5}$

Example: Rewrite the line $y = \frac{4}{7}x + 8$ in general form.

$7y = 4x + 56$

$0 = 4x - 7y + 56$

Example: Write the equation of a line perpendicular to 5x + 2y – 7 = 0 and with the same y-intercept as 7x – 6y + 1 = 0. Answer in general form.

$\frac{-2y = 5x - 7}{-2}$

$y = \frac{-5}{2}x + \frac{7}{2}$

$\perp \frac{2}{5}$

y-int $x=0$

$\cancel{7(0)} - 6y + 1 = 0$

$1 = 6y$

$y = \frac{1}{6}$

$\left(y = \frac{2}{5}x + \frac{1}{6}\right)30$

$30y = 12x + 5$

$0 = 12x - 30y + 5$

Slope-Point Form: $y - y_1 = m(x - x_1)$ where m is the slope of a line and x_1, y_1 is a coordinate.

Example: Determine the equation of a line with the points (4, 2) and (-1, 7).

$m = \frac{7-2}{-1-4} = \frac{5}{-5} = -1$

$y - 2 = -(x - 4)$

OR

$y - 7 = -1(x + 1)$

Example: Determine the slope and coordinate of the line y + 7 = 2(x – 4).

$m = 2$ point $(4, -7)$

Example: Find the equation, in general form, of the line perpendicular to the line 9x – 3y + 5 = 0 and same x-intercept as the line 4x – 3y – 3 = 0.

$\frac{3y = 9x + 5}{3}$

$y = 3x + \frac{5}{3}$

$\perp -\frac{1}{3}$

$\underline{x\text{int}}$ $y = 0$

$4x - 3\cancel{(0)} - 3 = 0$

$4x = 3$

$x = \frac{3}{4}$ $\left(\frac{3}{4}, 0\right)$

$y - 0 = \frac{-1}{3}\left(x - \frac{3}{4}\right)$

$\left(y = -\frac{1}{3}x + \frac{1}{4}\right) \cdot -12$

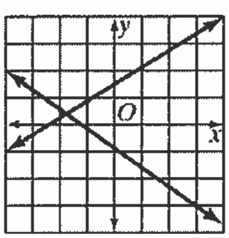
$-12y = 4x - 3$

$0 = 4x + 12y - 3$

Rate of Change: slope → can be increasing or decreasing ↗ ↘

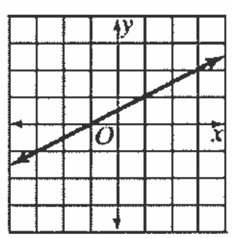
Unit 10: Systems of Equations

Intersecting Lines



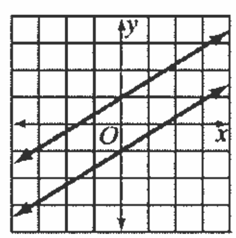
one solution

Coinciding Lines



no unique solution

Parallel Lines



no solution

Graphing: isolate y → graph y_1 and y_2 →find intersect
Substitution: isolate one variable → substitute the solution into the other equation → solve for the single variable → substitute that value into original equation to determine value of other variable

Elimination: multiply either one or both equations by values to get one of the variables either same or opposite coefficients → add or subtract to eliminate that variable → solve for the single variable → substitute that value into the original equation to solve for the other unknown variable

Example: Solve the following systems of equations.

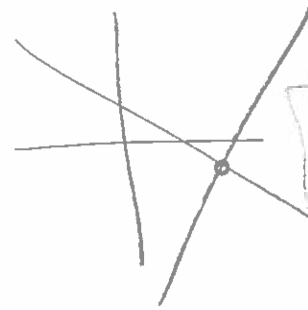
$2x$ $\quad 2x + 3y = 4$

$4x - y = 22$

Solve graphically:

$\frac{3y = -2x + 4}{3}$ $\quad$ $\frac{-y = -4x + 22}{-1}$

$y_1 = -\frac{2}{3}x + \frac{4}{3}$ $\quad$ $y_2 = 4x - 22$



$\boxed{(5, -2)}$

Solve algebraically:

$$\begin{array}{r} 4x + 6y = 8 \\ -\quad 4x - y = 22 \\ \hline 7y = -14 \end{array}$$

$\frac{7y = -14}{7}$ $\quad y = -2$

$4x + (+2) = 22$

$-2 \quad -2$

$\frac{4x = 20}{4}$ $\quad x = 5$

$\boxed{(5, -2)}$

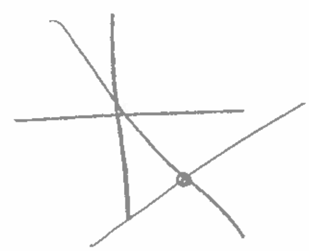
$5a + 3b = 3$ (x, y)

$3a - 7b = 81$ (x, y)

Graphically:

$\frac{3b = -5a + 3}{3}$ $\quad$ $\frac{-7b = -3a + 81}{-7}$

$b = -\frac{5}{3}a + 1$ $\quad$ $b = \frac{3}{7}a - \frac{81}{7}$



$\boxed{(6, -9)}$

Algebraically:

$(5a + 3b = 3)\,3$

$(3a - 7b = 81)\,5$

$$\begin{array}{r} 15a + 9b = 9 \\ -\ 15a - 35b = 405 \\ \hline 44b = -396 \end{array}$$

$\frac{44b = -396}{44}$

$b = -9$

$5a + 3(-9) = 3$

$5a - 27 = 3$

$\frac{5a = 30}{5}$

$a = 6$

$\boxed{(6, -9)}$

Example: Solve the following system of equations using the method of your choice.

4x + 2y – 13 = 0
3x = 5y +26

$(4x+2y=13)3$
$(3x-5y=26)4$

$\boxed{(4.5, -2.5)}$

$12x+6y=39$
$-\ 12x-20y=104$
$26y=-65$
$\overline{26}$
$y=-2.5$

$3x=5(-2.5)+26$
$3x=13.5$
$\overline{3}$
$x=4.5$

Example: Eli has a part-time job at the Snack Shack. On Saturday she sold 76 cones and 49 drinks for total revenue of \$179.55. On Sunday, she sold 54 cones and 37 drinks for total revenue of \$129.65. Find the price of each item.

c = \$ cone
d = \$ drink

$(76c+49d=179.55)37$
$(54c+37d=129.65)49$

$2812c+\cancel{1813d}=6643.35$
$-\ 2646c+\cancel{1813d}=6352.85$
$166c=290.5$
$\overline{166}$

$54(1.75)+37d=129.65$
$-94.5 \quad -94.5$
$37d=35.15$
$\overline{37}$

$\boxed{d=\$0.95/\text{drink}}$ $\boxed{c=\$1.75/\text{cone}}$

Example: Movie tickets are priced at \$8 for adults and \$4 for children. If 600 tickets were sold for a movie and the total amount of money collected was \$4000, how many tickets of each type were purchased?

a = # of adult
c = # of children

$8a+4c=4000$
$(a+c=600)4$

$8a+\cancel{4c}=4000$
$-\ 4a+\cancel{4c}=2400$
$4a=1600$
$\overline{4}$
$\boxed{a=400 \text{ adults}}$

$400+c=600$
$-400 \quad -400$
$\boxed{c=200 \text{ children}}$