Roots and Powers | Math 10C | Date: __________

Mixed and Entire Radicals Notes

Radicals are just numbers. As such, they can be added, subtracted, multiplied and divided. In this section, we will look at different ways to write a radical. We will use this fact to change radicals from one form to another, just like we can change fractions to decimals and back.

Equivalent expressions for any number have the same value. Equivalent means the same or equal.

$\sqrt{16\bullet 9}$ is equivalent to $\sqrt{16}\bullet\sqrt{9}$ because

$\sqrt{144}$ $\quad$ $4\bullet 3$

12 $\quad$ 12

Similarly,

$\sqrt[3]{8\bullet 27}$ is equivalent to $\sqrt[3]{8}\bullet\sqrt[3]{27}$ because

$\sqrt[3]{216}$ $\quad$ $2\bullet 3$

6 $\quad$ 6

Shown above is an example of the

Multiplication Property of Radicals

In general: $\sqrt[n]{ab}=\sqrt[n]{a}\bullet\sqrt[n]{b}$

where n is a natural number, and a and b are real numbers

We can use this property to simplify square roots and cube roots that are not perfect squares or perfect cubes, but have factors that are perfect squares or perfect cubes.

Example 1 – Simplifying Radicals Using Prime Factorization

Simplify each radical using prime factors.

a) $\sqrt{63}$

$\sqrt{7} \cdot \sqrt{9}$

3

$3\sqrt{7}$

b) $\sqrt[3]{108}$

$\sqrt[3]{4} \cdot \sqrt[3]{27}$

3

$3\sqrt[3]{4}$

c) $\sqrt[4]{128}$

$\sqrt[4]{8} \cdot \sqrt[4]{16}$

2

$2\sqrt[4]{8}$

Another way of simplifying radicals is to use perfect square, cube, etc factors of the radicand. This is where that chart will really help. The next example will demonstrate:

$\sqrt{63}$

7×9

$\boxed{3 \times 3}$

$3\sqrt{7}$

$\sqrt[3]{108}$

2 54

2×27

3 9

3 3

$3\sqrt[3]{4}$

$\sqrt[4]{128}$

2 64

2×32

2×16

2×8

2×4

2×2

$2\sqrt[4]{8}$

x	x^2	x^3	x^4	x^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64		
5	25	125		
6	36			
7	49			
8	64			
9	81			
10	100	1000		
11	121			
12	144			
13	169			
14	196			
15	225			

Example 2 – Writing Radicals in Simplest Form

Write each radical in simplest form, if possible.

a) $\sqrt{20}$

$\sqrt{4} \times \sqrt{5}$

$2\sqrt{5}$

b) $\sqrt{50}$

$\sqrt{25} \times \sqrt{2}$

$5\sqrt{2}$

c) $\sqrt{80}$

$\sqrt{16} \cdot \sqrt{5}$

$4\sqrt{5}$

d) $\sqrt{108}$

$\sqrt{36} \times \sqrt{3}$

$6\sqrt{3}$

e) $\sqrt{27}$

$\sqrt{9} \cdot \sqrt{3}$

$3\sqrt{3}$

f) $\sqrt[3]{32}$

$\sqrt[3]{8} \cdot \sqrt[3]{4}$

$2\sqrt[3]{4}$

g) $\sqrt[3]{135}$

$\sqrt[3]{27} \cdot \sqrt[3]{5}$

$3\sqrt[3]{5}$

h) $\sqrt[4]{48}$

$\sqrt[4]{16} \cdot \sqrt[4]{3}$

$2\sqrt[4]{3}$

Radicals of the form $\sqrt[n]{x}$ such as $\sqrt{80}$, $\sqrt[3]{144}$ and $\sqrt[4]{162}$ are called **entire radicals.**

Radicals of the form $a\sqrt[n]{x}$ such as $4\sqrt{5}$, $2\sqrt[3]{18}$, and $3\sqrt[4]{2}$ are called **mixed radicals.**

Entire radicals were rewritten as mixed radicals in *Examples 1* and *2*.

Any number can be written as the square root of its square; for example,

$2 = \sqrt{2 \cdot 2}$, $3 = \sqrt{3 \cdot 3}$, and $4 = \sqrt{4 \cdot 4}$, and so on.

Similarly, any number can be written as the cube root of its cube,

$5 = \sqrt[3]{5 \cdot 5 \cdot 5}$

or the fourth root of its perfect fourth power.

$6 = \sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6}$

We use this strategy to write a mixed radical as an entire radical.

Example 3 - Writing Mixed Radicals as Entire Radicals

Write each mixed radical as an entire radical

a) $7\sqrt{3}$

$7\sqrt{3}$

$\sqrt{3 \times (7)^2}$

$\sqrt{3 \times 49}$

$\sqrt{147}$

b) $2\sqrt[3]{4}$

$3\sqrt[3]{4}$

$\sqrt[3]{4 \times (3)^3}$

$\sqrt[3]{108}$

c) $2\sqrt[5]{3}$

$2\sqrt[5]{3}$

$\sqrt[5]{3 \times (2)^5}$

$\sqrt[5]{96}$